1 Quadratic Equations and Inequalities

1.1 Square Root Method

Lecture 1

Example. Solve \( x^2 = 4 \).

Without any instruction, an obvious answer to this problem is \( x = 2 \). And, this is seen as

\[
\begin{align*}
  x^2 &= 4 \\
  \sqrt{x^2} &= \sqrt{4} \\
  x &= 2.
\end{align*}
\]

In the previous chapter, when using the Pythagorean Theorem to find sides of triangles, this was an adequate way to consider the problem; however, it is only partially correct. Recall from earlier chapters that a quadratic was solved by setting the equation to zero, factoring, then setting each factor to zero. This is still the correct way to solve the problem. Then,

\[
\begin{align*}
  x^2 &= 4 \\
  x^2 - 4 &= 0 \\
  (x + 2)(x - 2) &= 0 \\
  x + 2 &= 0 \text{ or } x - 2 &= 0 \\
  x &= -2, 2 \\
  \text{Or, } x &= \pm 2
\end{align*}
\]

The point is to remember that a second degree equation should have two solutions. The problem can be solved by factoring or by noting that the solution side of the equation will always result in a “\( \pm \)”. Thus,

\[
\begin{align*}
  x^2 &= 4 \\
  \sqrt{x^2} &= \sqrt{4} \text{ Applying } \sqrt{} \text{ to both sides of equation.} \\
  x &= \pm 2.
\end{align*}
\]

Solving a quadratic in this way is called solving by the square root method. This technique isolates a perfect square that contains the variable to one side of the equation and all non-variable values are isolated on the other side of the equation. It works when the side of the equation with the variable can be written as a perfect square and all constants are isolated on the other side of the equation.
A perfect square is a number that is written to the power of two. For example, \(x^2\) is a perfect square. The number \((x - 1)^2\) is a perfect square. The number \(x^2 + 4 = x^2 + 2^2\) is not a perfect square because it is not written as \((\text{something})^2\).

Once the equation has the variable written as a perfect square on one side of the equation, then the square root of both sides. The following problems are designed to illustrate this.

**Example.** Solve \(3x^2 - 5 = 16\).

**Solution.**

\[
3x^2 - 5 = 16 \\
3x^2 = 21 \quad \text{Working to isolate the perfect square } x^2. \\
x^2 = 7 \quad \text{Isolated the perfect square.} \\
\sqrt{x^2} = \sqrt{7} \\
x = \pm \sqrt{7}
\]

**Example.** Solve \(x^2 + 4 = 0\).

**Solution.**

\[
x^2 + 4 = 0 \\
x^2 = -4 \quad \text{Isolated the perfect square of } x^2. \\
\sqrt{x^2} = \sqrt{-4} \\
x = \pm 2i
\]

**Example.** Solve \((2x - 1)^2 = 1\).

**Solution.**

\[
(2x - 1)^2 = 1 \\
\sqrt{(2x - 1)^2} = \sqrt{1} \\
2x - 1 = \pm 1 \\
2x = 1 \pm 1 \\
x = \frac{1 \pm 1}{2} \\
x = \frac{1 + 1}{2} \quad \text{or} \quad x = \frac{1 - 1}{2} \\
x = 1 \quad \text{or} \quad x = 0
\]
Example. Solve \((4x - 3)^2 = -2\).

Solution.

\[
(4x - 3)^2 = -2 \\
\sqrt{(4x - 3)^2} = \sqrt{-2} \\
4x - 3 = \pm i\sqrt{2} \\
4x = 3 \pm i\sqrt{2} \\
x = \frac{3 \pm i\sqrt{2}}{4}
\]

This last answer can be left in the fractional form with the \(\pm\) or it can be written as two answers

\[
x = \frac{3 + i\sqrt{2}}{4} \quad \text{or} \quad x = \frac{3 - i\sqrt{2}}{4}.
\]

Q-1: 1–17

Practice Problems.

1. \(x^2 = 36\) 
2. \(x^2 - 5 = 0\) 
3. \(x^2 - 24 = 0\) 
4. \(x^2 + 16 = 0\) 
5. \(2x^2 - 9 = 1\) 
6. \(3x^2 + 24 = 0\) 
7. \((x + 6)^2 = 9\) 
8. \((x - 5)^2 = 16\) 
9. \((3x - 1)^2 = 25\) 
10. \((x + 2)^2 = 18\) 
11. \(y^2 - 13 = 0\) 
12. \(x^2 = 40\) 
13. \(3y^2 = 21\) 
14. \((z + 1)^2 = -9\) 
15. \((y + 7)^2 = -4\) 
16. \((y - 1)^2 = -4\) 
17. \((3x - 4)^2 = -8\)
Solutions.

1. $x = \pm 6$
2. $x \pm \sqrt{5}$
3. $x^2 = \pm 2\sqrt{6}$
4. $x = \pm 4i$
5. $x = \pm \sqrt{5}$
6. $x = \pm 2i\sqrt{2}$
7. $x = -3, -9$
8. $x = -1, 9$
9. $x = -\frac{2}{3}, 2$
10. $x = \pm 3\sqrt{2} - 2, \text{ or } x = -2 \pm 3\sqrt{2}$
11. $y = \pm \sqrt{13}$
12. $x = \pm 2\sqrt{10}$
13. $y = \pm \sqrt{7}$
14. $z = -1 \pm 3i$
15. $y = \pm 2i - 7$
16. $y = 1 \pm 2i$
17. $x = \frac{4 \pm 2i\sqrt{2}}{3}$